ATMAM Mathematics Methods

Test 1 2018   Calculator Free

SHENTON COLLEGE

Name: ……………………………………………

Teacher: Friday   Smith

Time Allowed : 30 minutes

Marks	/31

Materials allowed: Formula Sheet.

Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given as exact values.
Marks may not be awarded for untidy or poorly arranged work.

1. **[2,2,2,2]**

   Differentiate each of the following with respect to x, clearly showing appropriate rules Do not simplify answers.

(a) $y = \frac{1}{2}x^3 - \frac{2}{x^2} + 5$

(b) $y = \frac{\cos x}{x^4 + 2}$

(c) $y = \sqrt{3x^2 + 4}$

(d) $y = e^{-x}\sin x$

2. [1,1]

Evaluate each of the following limits.

(a) $\lim_{h\to 0} \frac{e^h-1}{h}$

(b) $\lim_{h\to 0}\left(\frac{\cos(x+h)-\cos x}{h}\right)$

3. [3.1]

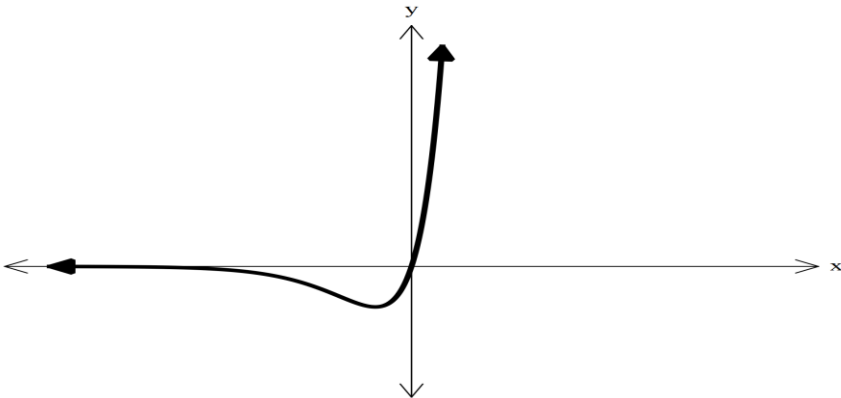Determine the value of $f''(-1)$ if $f(x) = (2x + 1)^5$.

Describe and explain the concavity of the curve at this point.

4 [4]

Find the equation of the tangent to $y = 3 - \sin(1 - 2x)$ at the point where $x = \frac{1}{2}$.

5. [3,2,3]

The graph of $y = f(x)$ is shown below, where $f(x) = 2xe^x$

(a) Determine the exact location of the stationary point on the graph of $y = f(x)$.

(b) Apply the second derivative test to show that the stationary point in (a) is a minimum.

(c) The graph of $y = f(x)$ has just one point of inflection. Determine the exact coordinates of this point.

6 [5]

Given the graph of $y = f'(x)$ provide possible graphs of $y = f(x)$ and $y = f''(x)$

[Care should be taken with the x values of critical points, but the 'heights' of the derivatives are not unique, use whatever makes your sketch easier to draw.]

f(x)

x

f '(x)

x

f "(x)

x
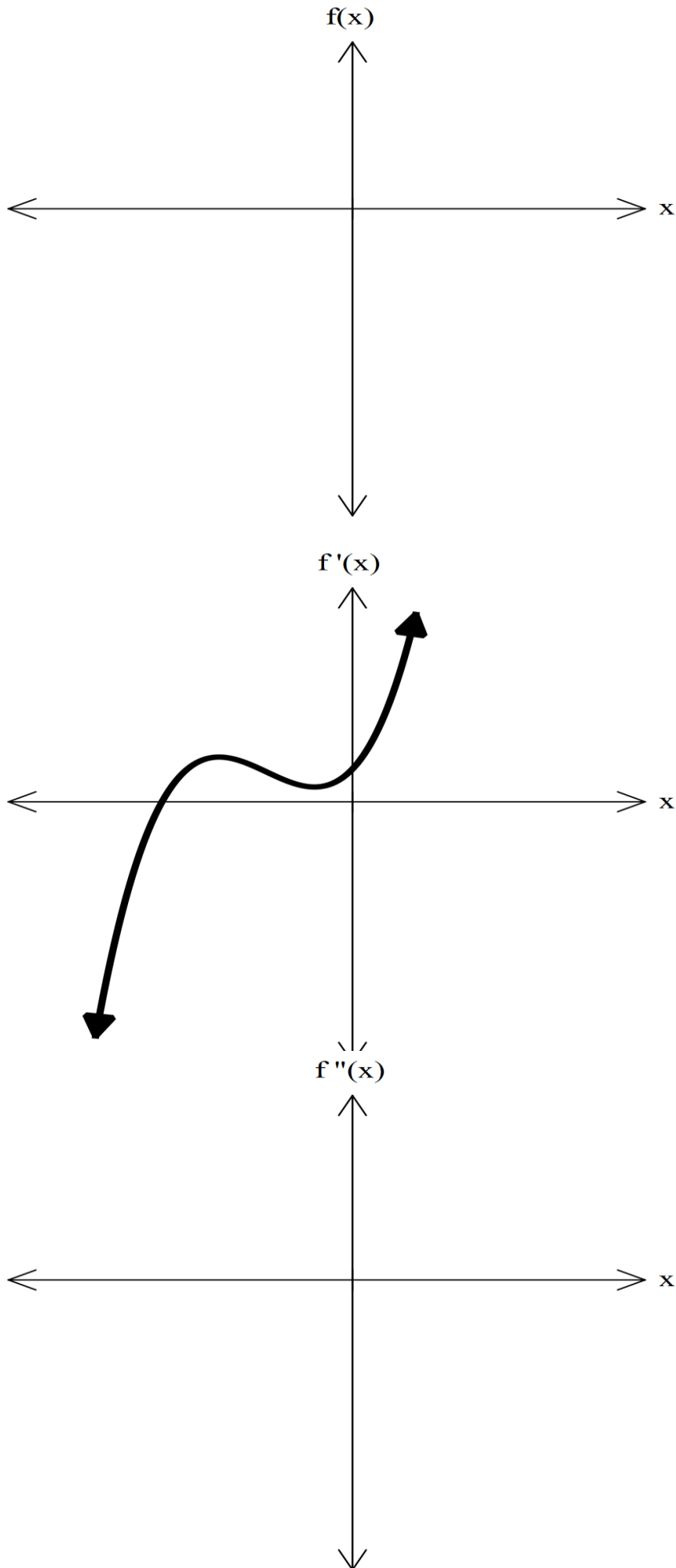
ATMAM Mathematics Methods

Test 1 2018 — Calculator Assumed

Name: …………………………………………………

Teacher: Friday   Smith

Time Allowed : 20 minutes   Marks   /19

Materials allowed: Classpad, calculator, formula sheet.

Attempt all questions.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given to <u>two</u> decimal places.
Marks may not be awarded for untidy or poorly arranged work.

7. **[1,1,1,1]**

The number of bees in a hive after t months is modelled by $B(t) = \dfrac{3000}{1+0.5e^{-1.73t}}.$

Determine:

(a) Determine the initial bee population.

(b) Determine the percentage increase in its population after one month.

(c) Explain why the population is increasing over time.

(d) Determine the rate at which the population is increasing after 3 months.

8. [1,1,1,2,4]

On the Indonesian coast, the depth of water t hours after midnight s given by $D(t) = 9.3 + 6.8\cos(0.507t)$ metres $0 \le t \le 24$

(a) Find the depth of the water at 8 am.

(b) Determine the maximum height of the water during this time.

(c) At what rate is the water changing at 8 am?

(d) At what time of day is water rising at its fastest rate?

(e) Show **how** to use calculus to determine the time(s) of day the height increasing at 1.5 metres per hour. Use your calculator to help you determine the time(s).

9. [1,1,1,1,1,1]

The population of a city over t years is given by $P = 120\ 000e^{0.07t}$

(a) Determine the population after 10 years.

(b) Find how long it takes for the population to double in size.

(c) Express the rate of growth as a function of t.

(d) Determine the rate of growth after 10 years.

(e) Express the rate of growth as a function of P

(f) Determine the growth rate when the Population is 3 million.

End of Questions